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## ABSTRACT

Many distance based indices of a graph have been studied in the literature. In this study, we introduce the multiplicative first and second status neighborhood indices, multiplicative sum and product connectivity status neighborhood indices, general multiplicative first and second status neighborhood indices, multiplicative  $F$ -status neighborhood index, general multiplicative status neighborhood index of a graph and compute exact formulas for some standard graphs and friendship graphs.

**KEYWORDS:** distance, status, multiplicative status neighborhood indices, multiplicative sum and product connectivity status neighborhood indices, multiplicative  $F$ -status neighborhood index, graph.

**Mathematics Subject Classification:** 05C05, 05C12, 05C35..

## 1. INTRODUCTION

We consider only finite, simple, connected graphs. Let  $G$  be a graph with vertex set  $V(G)$  and edge set  $E(G)$ . The degree  $d_G(u)$  of a vertex  $u$  is the number of vertices adjacent to  $u$ . The distance  $d(u, v)$  between any two vertices  $u$  and  $v$  is the length of shortest path connecting  $u$  and  $v$ . The status  $\sigma(u)$  of a vertex  $u$  in a connected graph  $G$  is the sum of its distance from every other vertex of  $G$ . Let  $N(v) = N_G(v) = \{u: uv \in E(G)\}$ . Let  $\sigma_n(v) = \sum_{u \in N(v)} \sigma(u)$  be the status sum of neighbor vertices. We refer [1] for graph theory theoretic terminologies.

A graph index or topological index is a numerical parameter mathematically derived from the graph structure [2]. Graph indices have their applications in various disciplines of Science and Technology, see [3, 4]. Many distance based indices of a graph such as Wiener index [5], Harary index [6], vertex status index [7] have been appeared in the literature. In this study, we propose some new multiplicative status neighborhood indices of a graph.

In [8], Kulli introduced the multiplicative first and second status indices of a graph, defined as

$$SII_1(G) = \prod_{uv \in E(G)} [\sigma(u) + \sigma(v)], \quad SII_2(G) = \prod_{uv \in E(G)} \sigma(u)\sigma(v).$$

Recently some variants of status indices were studied in [9, 10, 11, 12, 13, 14, 15, 16].

Motivated by the work on distance based multiplicative status indices, we introduce the multiplicative status neighborhood indices as follows:

The multiplicative first and second status neighborhood indices of a graph  $G$  are defined as

$$SN_1II(G) = \prod_{uv \in E(G)} [\sigma_n(u) + \sigma_n(v)], \quad SN_2II(G) = \prod_{uv \in E(G)} \sigma_n(u)\sigma_n(v).$$

The multiplicative first and second hyper status neighborhood indices of a graph  $G$  are defined as

$$HSN_1II(G) = \prod_{uv \in E(G)} [\sigma_n(u) + \sigma_n(v)]^2, \quad HSN_2II(G) = \prod_{uv \in E(G)} [\sigma_n(u)\sigma_n(v)]^2.$$

The modified multiplicative first and second status neighborhood indices of a graph  $G$  are defined as

$${}^m SN_1 II(G) = \prod_{uv \in E(G)} \frac{1}{\sigma_n(u) + \sigma_n(v)}, \quad {}^m SN_2 II(G) = \prod_{uv \in E(G)} \frac{1}{\sigma_n(u)\sigma_n(v)}.$$

Motivated by the work on multiplicative connectivity indices [4], we propose the following connectivity status neighborhood indices:

The multiplicative sum and product connectivity status neighborhood indices of a graph  $G$  are defined as

$$SSNII(G) = \prod_{uv \in E(G)} \frac{1}{\sqrt{\sigma_n(u) + \sigma_n(v)}}, \quad PSNII(G) = \prod_{uv \in E(G)} \frac{1}{\sqrt{\sigma_n(u)\sigma_n(v)}}.$$

The reciprocal product connectivity status neighborhood index of a graph  $G$  is defined as

$$RPSNII(G) = \prod_{uv \in E(G)} \sqrt{\sigma_n(u)\sigma_n(v)}.$$

The general multiplicative first and second status neighborhood indices of a graph  $G$  are defined as

$$SN_1^a II(G) = \prod_{uv \in E(G)} [\sigma_n(u) + \sigma_n(v)]^a,$$

$$SN_2^a II(G) = \prod_{uv \in E(G)} [\sigma_n(u)\sigma_n(v)]^a,$$

where  $a$  is a real number.

The multiplicative  $F$ -status neighborhood index of a graph  $G$  is defined as

$$FSNII(G) = \prod_{uv \in E(G)} [\sigma_n(u)^2 + \sigma_n(v)^2].$$

The general multiplicative status neighborhood index of a graph  $G$  is defined as

$$SN^a II(G) = \prod_{uv \in E(G)} [\sigma_n(u)^a + \sigma_n(v)^a].$$

We introduce the multiplicative inverse sum status neighborhood index of a graph  $G$ , and it is defined as

$$ISNII(G) = \prod_{uv \in E(G)} \frac{\sigma_n(u)\sigma_n(v)}{\sigma_n(u) + \sigma_n(v)}.$$

We propose the multiplicative harmonic status neighborhood index of a graph  $G$ , defined as

$$HSNII(G) = \prod_{uv \in E(G)} \frac{2}{\sigma_n(u) + \sigma_n(v)}.$$

In this paper, newly defined multiplicative status neighborhood indices for complete graphs, wheel graphs, friendship graphs are determined.

## 2. RESULTS FOR COMPLETE GRAPHS

In the following theorem, we compute the general multiplicative first status neighborhood index of a complete graph  $K_n$ .

**Theorem 1.** The general multiplicative first status neighborhood index of a complete graph  $K_n$  is

$$SN_1^a II(K_n) = [2(n-1)^2]^{\frac{1}{2}an(n-1)}. \quad (1)$$

**Proof:** Let  $K_n$  be a complete graph with  $n$  vertices and  $\frac{n(n-1)}{2}$  edges. For any vertex  $u$  of  $K_n$ ,  $\sigma(u) = n-1$ . Clearly for any vertex  $u$  of  $K_n$ ,  $\sigma_n(u) = (n-1)^2$ . Thus

$$\begin{aligned} SN_1^a II(K_n) &= \prod_{uv \in E(K_n)} [\sigma_n(u)^a + \sigma_n(v)^a] = [(n-1)^2 + (n-1)^2]^{\frac{1}{2}an(n-1)} \\ &= [2(n-1)^2]^{\frac{1}{2}an(n-1)}. \end{aligned}$$

We obtain the following results by using Theorem 1.

**Corollary 1.1.** The multiplicative first status neighborhood index of  $K_n$  is

$$SN_1 II(K_n) = [2(n-1)^2]^{\frac{1}{2}n(n-1)}.$$

**Corollary 1.2.** The multiplicative first hyper status neighborhood index of  $K_n$  is

$$HSN_1 II(K_n) = [2(n-1)^2]^{n(n-1)}.$$

**Corollary 1.3.** The modified multiplicative first status neighborhood index of  $K_n$  is

$${}^m SN_1 II(K_n) = [2(n-1)^2]^{\frac{1}{2}n(n-1)}.$$

**Corollary 1.4.** The multiplicative sum connectivity status neighborhood index of  $K_n$  is

$$SSN II(K_n) = [2(n-1)^2]^{\frac{1}{4}n(n-1)}.$$

**Proof:** Put  $a = 1, 2, -1, -\frac{1}{2}$  in equation (1), we obtain the desired results.

In the following theorem, we determine the general multiplicative second status neighborhood index of complete graph  $K_n$ .

**Theorem 2.** The general multiplicative second status neighborhood index of a complete graph  $K_n$  is

$$SN_2^a II(K_n) = (n-1)^{2an(n-1)}. \quad (2)$$

**Proof:** Let  $K_n$  be a complete graph, then  $\sigma_n(u) = (n-1)^2$  for any vertex  $u$  of  $K_n$ . Therefore

$$\begin{aligned} SN_2^a II(K_n) &= \prod_{uv \in E(K_n)} [\sigma_n(u)^a \sigma_n(v)^a] = [(n-1)^{2a} (n-1)^{2a}]^{\frac{1}{2}n(n-1)} \\ &= (n-1)^{2an(n-1)}. \end{aligned}$$

From Theorem 2, we obtain the following results.

**Corollary 2.1.** The multiplicative second status neighborhood index of  $K_n$  is

$$SN_2 II(K_n) = (n-1)^{2n(n-1)}.$$

**Corollary 2.2.** The multiplicative second hyper status neighborhood index of  $K_n$  is

$$HSN_2 II(K_n) = (n-1)^{4n(n-1)}.$$

**Corollary 2.3.** The modified multiplicative second status neighborhood index of  $K_n$  is

$${}^m SN_2 II(K_n) = \left(\frac{1}{n-1}\right)^{2n(n-1)}.$$

**Corollary 2.4.** The multiplicative product connectivity status neighborhood index of  $K_n$  is

$$PSNII(K_n) = \left(\frac{1}{n-1}\right)^{n(n-1)}.$$

**Corollary 2.5.** The multiplicative reciprocal product connectivity status neighborhood index of  $K_n$  is

$$RPSNII(K_n) = (n-1)^{n(n-1)}.$$

**Proof:** Put  $a = 1, 2, -1, -1/2, 1/2$  in equation (2), we get the desired results.

In the following theorem, we compute the general multiplicative status neighborhood index of a complete graph  $K_n$ .

**Theorem 3.** The general multiplicative status neighborhood index of a complete graph  $K_n$  is

$$SN^a II(K_n) = [2(n-1)^{2a}]^{\frac{1}{2}n(n-1)}. \quad (2)$$

**Proof:** Let  $K_n$  be a complete graph. Then  $\sigma_n(u) = (n-1)^2$  for any vertex  $u$  of  $K_n$ . Thus

$$\begin{aligned} SN^a II(K_n) &= \prod_{uv \in E(K_n)} [\sigma_n(u)^a + \sigma_n(v)^a] = [(n-1)^{2a} + (n-1)^{2a}]^{\frac{1}{2}n(n-1)} \\ &= [2(n-1)^{2a}]^{\frac{1}{2}n(n-1)}. \end{aligned}$$

We obtain the following result from Theorem 3.

**Corollary 3.1.** The multiplicative  $F$ -status neighborhood index of  $K_n$  is

$$FSNII(K_n) = [2(n-1)^4]^{\frac{1}{2}n(n-1)}.$$

In the following theorem, we compute the general multiplicative inverse sum status neighborhood index of  $K_n$ .

**Theorem 4.** The multiplicative inverse sum status neighborhood index of a complete graph  $K_n$  is

$$ISNII(K_n) = \left[\frac{1}{2}(n-1)^2\right]^{\frac{1}{2}n(n-1)}.$$

**Proof:** From definition and  $\sigma_n(u) = (n-1)^2$  for any vertex  $u$  of  $K_n$ , we deduce

$$\begin{aligned} ISNII(K_n) &= \prod_{uv \in E(K_n)} \frac{\sigma_n(u)\sigma_n(v)}{\sigma_n(u) + \sigma_n(v)} = \left[\frac{(n-1)^2(n-1)^2}{(n-1)^2 + (n-1)^2}\right]^{\frac{1}{2}n(n-1)} \\ &= \left[\frac{1}{2}(n-1)^2\right]^{\frac{1}{2}n(n-1)}. \end{aligned}$$

**Theorem 5.** The multiplicative harmonic status neighborhood index of  $K_n$  is

$$HSNII(K_n) = \frac{1}{(n-1)^{n(n-1)}}.$$

**Proof:** We have  $\sigma_n(u) = (n-1)^2$  for any vertex  $u$  of  $K_n$ . Thus

$$HSNII(K_n) = \prod_{uv \in E(K_n)} \frac{2}{\sigma_n(u) + \sigma_n(v)} = \left[\frac{2}{(n-1)^2 + (n-1)^2}\right]^{\frac{1}{2}n(n-1)}.$$

$$= \frac{1}{(n-1)^{n(n-1)}}.$$

### 3. RESULTS FOR WHEEL GRAPHS

A wheel graph  $W_n$  is the join of  $K_1$  and  $C_n$ . A graph  $W_4$  is shown in Figure 1.

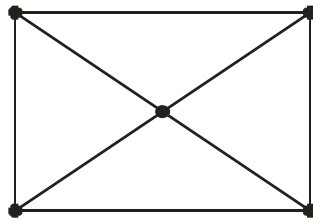


Figure 1. Wheel graph  $W_4$

A graph  $W_n$ , has  $n+1$  vertices and  $2n$  edges. In a graph  $W_n$ , there are two types of status edges as follows:

$$E_1 = \{uv \in E(W_n) \mid \sigma(u) = \sigma(v) = 2n - 3\}, \quad |E_1| = n.$$

$$E_2 = \{uv \in E(W_n) \mid \sigma(u) = n, \sigma(v) = 2n - 3\}, \quad |E_2| = n.$$

Thus by calculation, we obtain that there are two types of status neighborhood edges as given in Table 1.

Table 1. Status neighborhood edge partition of  $W_n$

|  |                    |                       |
|--|--------------------|-----------------------|
| $\sigma_n(u), \sigma_n(v) \setminus uv \in E(W_n)$ | $(5n - 6, 5n - 6)$ | $(5n - 6, n(2n - 3))$ |
| Number of vertices                                 | $n$                | $n$                   |

In the following theorem, we compute the general multiplicative first status neighborhood index of a wheel graph  $W_n$ .

**Theorem 6.** The general multiplicative first status neighborhood index of  $W_n$  is given by

$$SN_1^a II(W_n) = (10n - 12)^{an} \times (2n^2 + 2n - 6)^{an}. \quad (3)$$

**Proof:** From definition and by using Table 1, we deduce

$$SN_1^a(W_n) = \prod_{uv \in E(W_n)} [\sigma_n(u) + \sigma_n(v)]^a$$

$$= (5n - 6 + 5n - 6)^{an} \times (5n - 6 + 2n^2 - 3n)^{an}$$

$$= (10n - 12)^{an} \times (2n^2 + 2n - 6)^{an}$$

We obtain the following results from Theorem 6.

**Corollary 6.1.** The multiplicative first status neighborhood index of  $W_n$  is

$$SN_1 II(W_n) = 2^{2n} (5n - 6)^n (n^2 + n - 3)^n.$$

**Corollary 6.2.** The multiplicative first hyper status neighborhood index of  $W_n$  is

$$HSN_1 II(W_n) = 2^{4n} (5n - 6)^{2n} (n^2 + n - 3)^{2n}.$$

**Corollary 6.3.** The modified multiplicative first status neighborhood index of  $W_n$  is

$${}^m SN_1 II(W_n) = \left(\frac{1}{4}\right)^n \times \left(\frac{1}{5n-6}\right)^n \times \left(\frac{1}{n^2+n-3}\right)^n.$$

**Corollary 6.4.** The multiplicative sum connectivity status neighborhood index of  $W_n$  is

$$SSNII(W_n) = \left(\frac{1}{2}\right)^n \times \left(\frac{1}{5n-6}\right)^{\frac{n}{2}} \times \left(\frac{1}{n^2+n-3}\right)^{\frac{n}{2}}.$$

**Proof:** Put  $a = 1, 2, -1, -\frac{1}{2}$  in equation (3), we obtain the desired results.

In the following theorem, we compute the general multiplicative second status neighborhood index of a wheel graph  $W_n$ .

**Theorem 7.** The general second status neighborhood index of  $W_n$  is

$$SN_2^a II(W_n) = (5n-6)^{2an} \times (10n^3 - 27n^2 + 18n)^{an}. \quad (4)$$

**Proof:** By definition and by using Table 1, we derive

$$\begin{aligned} SN_2^a II(W_n) &= \prod_{uv \in E(W_n)} [\sigma_n(u)\sigma_n(v)]^a \\ &= [(5n-6)(5n-6)]^{an} \times [(5n-6)(2n^2-3n)]^{an} \\ &= (5n-6)^{2an} \times (10n^3 - 27n^2 + 18n)^{an}. \end{aligned}$$

We obtain the following results by using Theorem 7.

**Corollary 7.1.** The multiplicative second status neighborhood index of  $W_n$  is

$$SN_2 II(W_n) = (5n-6)^{2n} \times (10n^3 - 27n^2 + 18n)^n.$$

**Corollary 7.2.** The multiplicative second hyper status neighborhood index of  $W_n$  is

$$HSN_2 II(W_n) = (5n-6)^{4n} \times (10n^3 - 27n^2 + 18n)^{2n}.$$

**Corollary 7.3.** The modified multiplicative second status neighborhood index of  $W_n$  is

$${}^m SN_2 II(W_n) = \frac{1}{(5n-6)^{2n}} \times \frac{1}{(10n^3 - 27n^2 + 18n)^n}.$$

**Corollary 7.4.** The multiplicative product connectivity status neighborhood index of  $W_n$  is

$$PSNII(W_n) = \frac{1}{(5n-6)^n} \times \frac{1}{(10n^3 - 27n^2 + 18n)^{\frac{n}{2}}}.$$

**Corollary 7.5.** The multiplicative reciprocal product connectivity status neighborhood index of  $W_n$  is

$$RPSNII(W_n) = (5n-6)^n \times (10n^3 - 27n^2 + 18n)^{\frac{n}{2}}.$$

**Proof:** Put  $a = 1, 2, -1, -\frac{1}{2}, \frac{1}{2}$  in equation (4), we obtain the desired results.

In the following theorem, we compute the general multiplicative status neighborhood index of a wheel graph  $W_n$ .

**Theorem 8.** The general multiplicative status neighborhood index of  $W_n$  is

$$SN^a II(W_n) = 2^n (5n-6)^{an} \times [(5n-6)^a + (2n^2-3n)^a]^n.$$

**Proof:** Let  $K_n$  be a complete graph. Then  $\sigma_n(u) = (n-1)^2$  for any vertex  $u$  of  $K_n$ . Thus

$$\begin{aligned}
 SN^a II(W_n) &= \prod_{uv \in E(W_n)} [\sigma_n(u)^a + \sigma_n(v)^a] \\
 &= [(5n-6)^a + (5n-6)^a]^n \times [(5n-6)^a + (2n^2-3n)^a]^n \\
 &= 2^n (5n-6)^{an} \times [(5n-6)^a + (2n^2-3n)^a]^n.
 \end{aligned}$$

We establish the following results by using Theorem 8.

**Corollary 8.1.** The multiplicative  $F$ -status neighborhood index of  $W_n$  is

$$FSNII(W_n) = 4(5n-6)^{2n} \times (4n^4 - 12n^3 + 34n^2 - 60n + 36)^n.$$

In the following theorem, we compute the multiplicative inverse sum status neighborhood index of  $W_n$ .

**Theorem 9.** The multiplicative inverse sum status neighborhood index of a wheel graph  $W_n$  is

$$ISNII(W_n) = \frac{1}{2^{2n}} (5n-6)^n \left[ \frac{10n^3 - 27n^2 + 18n}{n^2 + n - 3} \right]^n.$$

**Proof:** From definition and by using Table 1, we deduce

$$\begin{aligned}
 ISNII(W_n) &= \prod_{uv \in E(K_n)} \frac{\sigma_n(u)\sigma_n(v)}{\sigma_n(u) + \sigma_n(v)} \\
 &= \left[ \frac{(5n-6)(5n-6)}{(5n-6) + (5n-6)} \right]^n \times \left[ \frac{(5n-6)(2n^2-3n)}{5n-6 + 2n^2-3n} \right]^n \\
 &= \frac{1}{2^{2n}} (5n-6)^n \left[ \frac{10n^3 - 27n^2 + 18n}{n^2 + n - 3} \right]^n.
 \end{aligned}$$

In the following theorem, we determine the multiplicative harmonic status neighborhood index of  $W_n$ .

**Theorem 10.** The multiplicative harmonic status neighborhood index of a wheel graph  $W_n$  is

$$HSNII(W_n) = \frac{1}{(5n-6)^n (n^2 + n - 3)^n}.$$

**Proof:** From definition and by using Table 1, we derive

$$\begin{aligned}
 HSNII(W_n) &= \prod_{uv \in E(W_n)} \frac{2}{\sigma_n(u) + \sigma_n(v)} \\
 &= \left( \frac{2}{5n-6 + 5n-6} \right)^n \times \left( \frac{2}{5n-6 + 2n^2-3n} \right)^n \\
 &= \frac{1}{(5n-6)^n (n^2 + n - 3)^n}.
 \end{aligned}$$

#### 4. RESULTS FOR FRIENDSHIP GRAPHS

A friendship graph is the graph obtained by taking  $n \geq 2$  copies of  $C_3$  with vertex in common and is denoted by  $F_n$ . The graph  $F_4$  is shown in Figure 2.



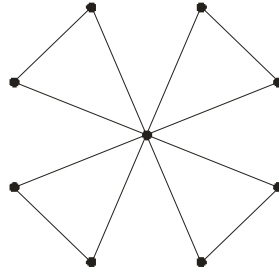


Figure 2. Friendship graph  $F_4$

A graph  $F_n$  has  $2n+1$  vertices and  $3n$  edges. By calculation, we obtain that there are two types of status edges as follows:

$$E_1 = \{uv \in E(F_n) \mid \sigma(u) = \sigma(v) = 2n - 3\}, \quad |E_1| = n.$$

$$E_2 = \{uv \in E(F_n) \mid \sigma(u) = 2n, \sigma(v) = 4n - 2\}, \quad |E_2| = 2n.$$

By calculation, we have two types of status neighborhood edges as given in Table 2.

Table 2. Status neighborhood edge partition of  $F_n$

| $\sigma_n(u), \sigma_n(v) \setminus uv \in E(F_n)$ | $(6n - 2, 6n - 2)$ | $(6n - 2, 2n(4n - 2))$ |
|--|--------------------|------------------------|
| Number of edges                                    | $n$                | $2n$                   |

**Theorem 11.** The general multiplicative first status neighborhood index of  $F_n$  is

$$SN_1^a II(F_n) = 4^{2an} (3n - 1)^{an} (4n^2 + n - 1)^{2an}. \tag{5}$$

**Proof:** From definition and by using Table 2, we derive

$$SN_1^a II(F_n) = \prod_{uv \in E(F_n)} [\sigma_n(u) + \sigma_n(v)]^a$$

$$= (6n - 2 + 6n - 2)^{an} (6n - 2 + 8n^2 - 4n)^{2an}$$

$$= 4^{2an} (3n - 1)^{an} (4n^2 + n - 1)^{2an}.$$

We obtain the following results from Theorem 11.

**Corollary 11.1.** The multiplicative first status neighborhood index of  $F_n$  is

$$SN_1 II(F_n) = 4^{2n} (3n - 1)^n (4n^2 + n - 1)^{2n}.$$

**Corollary 11.2.** The multiplicative first hyper status neighborhood index of  $F_n$  is

$$HSN_1 II(F_n) = 4^{4n} (3n - 1)^{2n} (4n^2 + n - 1)^{4n}.$$

**Corollary 11.3.** The modified multiplicative first status neighborhood index of  $F_n$  is

$${}^m SN_1 II(F_n) = \frac{1}{4^{2n} (3n - 1)^n (4n^2 + n - 1)^{2n}}.$$

**Corollary 11.4.** The multiplicative sum connectivity status neighborhood index of  $F_n$  is

$$SSN II(F_n) = \frac{1}{4^n (3n - 1)^{\frac{n}{2}} (4n^2 + n - 1)^n}.$$

**Proof:** Put  $a = 1, 2, -1, -\frac{1}{2}$  in equation (5), we get the desired results.

In the following theorem, we determine the general multiplicative second status neighborhood index of a friendship graph  $F_n$ .

**Theorem 12.** The general multiplicative second status neighborhood index of a friendship graph  $F_n$  is



$$SN_2^a II(F_n) = 2^{8an} (3n-1)^{4an} (2n^2 - n)^{2an}. \tag{6}$$

**Proof:** From definition and by using Table 2, we deduce

$$\begin{aligned} SN_2^a II(F_n) &= \prod_{uv \in E(F)} [\sigma_n(u) \sigma_n(v)]^a \\ &= [(6n-2)(6n-2)]^{an} \times [(6n-2)(8n^2 - 4n)]^{2an} \\ &= 2^{8an} (3n-1)^{4an} (2n^2 - n)^{2an}. \end{aligned}$$

We obtain the following results from Theorem 12.

**Corollary 12.1.** The multiplicative second status neighborhood index of  $F_n$  is

$$SN_2 II(F_n) = 2^{8n} (3n-1)^{4n} (2n^2 - n)^{2n}.$$

**Corollary 12.2.** The multiplicative second hyper status neighborhood index of  $F_n$  is

$$HSN_2 II(F_n) = 2^{16n} (3n-1)^{8n} (2n^2 - n)^{4n}.$$

**Corollary 12.3.** The modified multiplicative second status neighborhood index of  $F_n$  is

$${}^m SN_2 II(F_n) = \frac{1}{2^{8n} (3n-1)^{4n} (2n^2 - n)^{2n}}.$$

**Corollary 12.4.** The multiplicative product connectivity status neighborhood index of  $F_n$  is

$$PSNII(F_n) = \frac{1}{2^{4n} (3n-1)^{2n} (2n^2 - n)^n}.$$

**Corollary 12.5.** The multiplicative reciprocal product connectivity status neighborhood index of  $F_n$  is

$$RPSNII(F_n) = 2^{4n} (3n-1)^{2n} (2n^2 - n)^n.$$

**Proof:** Put  $a = 1, 2, -1, -\frac{1}{2}, \frac{1}{2}$  in equation (6), we get the desired results.

In the following theorem, we determine the general multiplicative status neighborhood index of a friendship graph  $F_n$ .

**Theorem 13.** The general multiplicative status neighborhood index of friendship graph  $F_n$  is

$$SN^a II(F_n) = [2(6n-2)^a]^n \times [(6n-2)^a + (8n^2 - 4n)^a]^{2n}.$$

**Proof:** From definition and by using Table 2, we deduce

$$\begin{aligned} SN^a II(F_n) &= \prod_{uv \in E(F_n)} [\sigma_n(u)^a + \sigma_n(v)^a] \\ &= [(6n-2)^a + (6n-2)^a]^n \times [(6n-2)^a + (8n^2 - 4n)^a]^{2n} \\ &= [2(6n-2)^a]^n \times [(6n-2)^a + (8n^2 - 4n)^a]^{2n}. \end{aligned}$$

We establish the following results from Theorem 13.

**Corollary 13.1.** The multiplicative  $F$ -status neighborhood index of  $F_n$  is

$$FSNII(F_n) = 8^{2n} (3n-1)^n \times [16n^4 - 16n^3 + 13n^2 - 6n + 1]^{2n}.$$

In the following theorem, we compute the multiplicative inverse sum status neighborhood index of  $F_n$ .

**Theorem 14.** The multiplicative inverse sum status neighborhood index of a friendship graph  $F_n$  is

$$ISNII(F_n) = 4^{2n} (3n-1)^n \left[ \frac{6n^3 - 5n^2 + n}{4n^2 + n - 1} \right]^{2n}.$$

**Proof:** Using definition and Table 2, we obtain

$$\begin{aligned} ISNII(F_n) &= \prod_{uv \in E(F_n)} \frac{\sigma_n(u)\sigma_n(v)}{\sigma_n(u) + \sigma_n(v)} \\ &= \left[ \frac{(6n-2)(6n-2)}{6n-2+6n-2} \right]^n \times \left[ \frac{(6n-2)(8n^2-4n)}{6n-2+8n^2-4n} \right]^{2n} \\ &= 4^{2n} (3n-1)^n \left[ \frac{6n^3 - 5n^2 + n}{4n^2 + n - 1} \right]^{2n}. \end{aligned}$$

In the following theorem, we determine the multiplicative harmonic status neighborhood index of  $F_n$ .

**Theorem 15.** The multiplicative harmonic status neighborhood index of a friendship graph  $F_n$  is

$$HSNII(F_n) = \frac{1}{(6n-2)^n (4n^2 + n - 1)^{2n}}.$$

**Proof:** Using definition and Table 2, we have

$$\begin{aligned} HSNII(F_n) &= \prod_{uv \in E(F_n)} \frac{2}{\sigma_n(u) + \sigma_n(v)} \\ &= \left( \frac{2}{6n-2+6n-2} \right)^n \times \left( \frac{2}{6n-2+8n^2-4n} \right)^{2n} \\ &= \frac{1}{(6n-2)^n (4n^2 + n - 1)^{2n}}. \end{aligned}$$

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